SCRAP PAPER GEOMETRY: PROPERTIES OF TRIANGLES

Wayne R. Scott
Genesee Intermediate School District
Flint, MI 48507

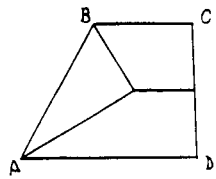
Only a small percent of our students enroll in a formal geometry course. If they do not experience geometry in an informal setting they have been deprived of some of the most valuable mathematical experiences they could have. Geometry is every bit as essential as the ability to compute. Several such activities are described in this article. The activities should be done as the article is read. The drawings have dotted lines where the fold will occur and solid lines for creases.

Prior to beginning collect some scrap paper, a marker and a compass (the drawing kind, not the north pole type). A folded sheet of paper serves as a straight edge. The drawings have been done assuming that the paper is 8 1/2 inches by 11 inches.

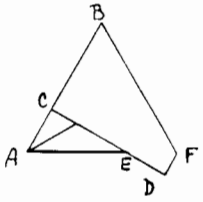
Activity 1:

Fold a sheet of paper in half lengthways.

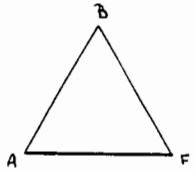
Open it and fold the top left vertex (the northwest corner) to the center crease with the fold beginning from the lower left vertex (southwest corner).



The angle at A appears to be 60 degrees but let's check. Bisect angle ABC (fold BC over BA).



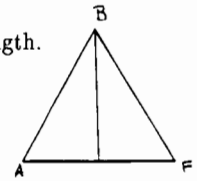
There is a small scrap DEF which needs to be folded up to show the triangular region ABF. Fold it up. Note that in doing this angle DFB is bisected. Flip the paper over.



It looks like an equilateral triangle. Since equilateral means equal sides, our hypothesis can easily be tested. Place BF over AB and crease.

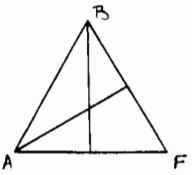
Note:

1. Angle B has been bisected.
2. The perpendicular bisector of AF has been folded.
3. This new crease is an altitude.
4. The new crease is also a median (A line from the vertex to the midpoint of the opposite side).
5. Angle F is congruent to angle A.
6. Sides BF and BA are the same length.

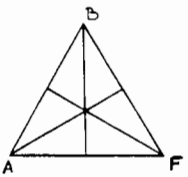


Open the triangle.

Fold AF over AB and crease. Note all of the new congruences.



Fold AF over BF and crease.



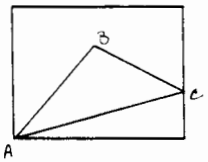
Note all of the congruences.

1. The sides are the same length thus the triangle is equilateral.
2. The angles are congruent so the triangle is equiangular.
3. The medians, altitudes, perpendicular bisectors of the sides, angle bisectors are all the same and they all meet at the same point.

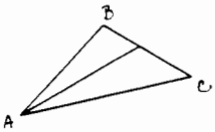
Activity 2: Do the angles bisectors of a triangle meet?

Crease another sheet of paper three times making a triangle on the interior of the sheet of paper.

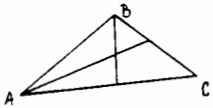
Cut it out.



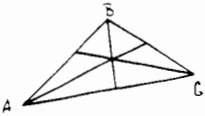
Bisect angle A



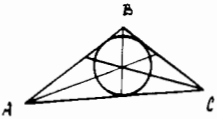
Bisect angle B



Bisect angle C



The angle bisectors all meet at one point. This point is called the incenter. With a compass draw a circle tangent to (just touching) one of the sides of the triangle with the incenter as its center. What happens with the other sides?

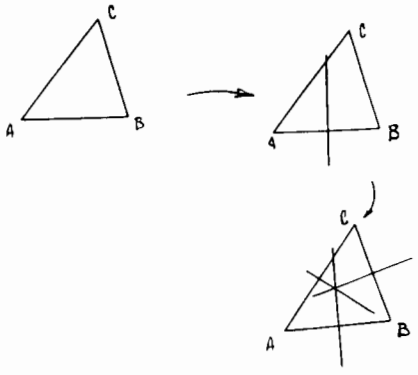


The circle is tangent to all three sides. This point, the incenter, is the center of the circle which is tangent to all three sides of the triangle.

Activity 3: Are there other special points?

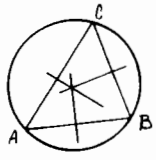
Using another sheet of paper, fold a new triangle. **Do not cut it out.** Crease the paper to get the perpendicular bisector of side AB.

Fold to get the perpendicular bisectors of BC and AC. All three perpendicular bisectors meet at a point. If this point is off the sheet of paper, place it on another sheet and extend the lines with a straight edge (ruler or a folded sheet of paper) being careful not to move the sheets of paper.



If these lines are drawn, use another color and draw them only from the side of the triangle to this point of intersection. Open the paper.

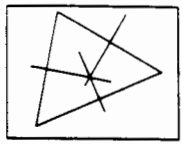
This new point, P, should be called the "outcenter" but it is not. It is called the circumcenter. Using P as the center, draw a circle (or as much of the circle as possible) which passes through B.



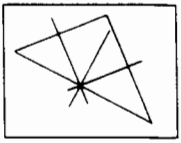
The circle goes through all three vertices, A, B and C. It "circumscribes" (goes around yet touches) the triangle. In our first example the intersection of the perpendicular bisectors (incenter) was inside the triangle. Will the incenter always be inside the triangle? (Yes).

In our example the intersection of the perpendicular bisectors (circumcenter) was inside the triangle. Is the circumcenter always inside the triangle? (No) Is it ever outside the triangle? (Yes) Experiment with different triangles to find out when the circumcenter is inside, on and outside the triangle.

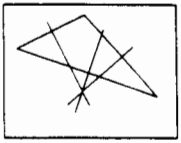
All the angles are less than 90 degrees (An acute triangle)



One of the angles is 90 degrees (A right triangle)



One of the angles is greater than 90 degrees (An obtuse triangle)

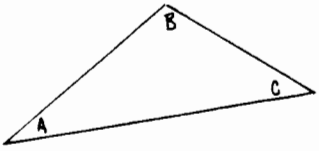


Sample Triangle Activity

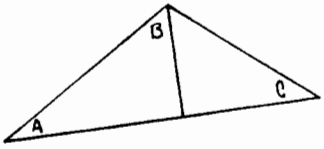
In this activity it is necessary to know that:

1. a line segment can be bisected by folding the end points together. (i.e. the midpoint)
2. a perpendicular can be obtained by folding a line back on itself.

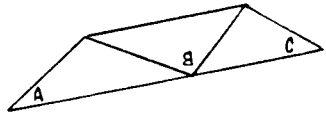
Begin with any triangle. Label the largest angle B



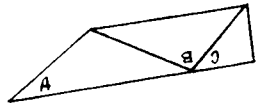
Fold the altitude to the largest vertex, B. (That is fold AC over itself so that the crease goes through vertex B.)



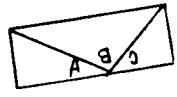
Fold vertex B down to the foot of the altitude.



Fold vertex C to the foot of the altitude.



Fold vertex A to the foot of the altitude.



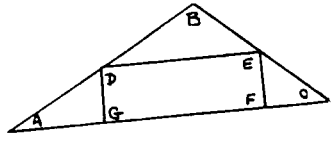
Now let's inspect our results.

The sum of the measures of angles A, B and C is _____ (a straight angle).

The shape of the new figure is _____ (a rectangle) to make the original triangle.

Open the triangle.

Mark points D, E, F and G.



The length of BE _____ the length of EC. (equals)

The length of AD _____ the length of DB. (equals)

D and E are midpoints of segments AB and BC respectively (DE joins the midpoints of two sides of the triangle)

Figure DEFG is a rectangle.
Therefore, DE and FG are
_____ (equal in length) and _____ (parallel).

But GF is half the length of AC
Therefore DE is parallel to AC and equal to half its length.
(The line joining the midpoints of 2 sides of a triangle is parallel to the third side and equal to half its length).

MATH SCRAMBLER

Unscramble these four mixed-up math terms, one letter to each blank:

E X I S S ☐ ___ ☐ ___ ☐

G A L E N ☐ ☐ ___ ___ ___

T R Y H I T ___ ___ ☐ ___ ___ ☐

G O B L O N ☐ ___ ___ ☐ ___ ___

Now, rearrange the letters in the boxes to form the answer to the question below:

DON'T YOU KIDS KNOW STRAIGHT UP?

___ ___, ___ ___ ___ – ___ ___ ___ ___?

Answer is on page 22.
